Massachusetts Institute of Technology
Department of Physics

Physics 8.07 Fall 2005

Problem Set #1
Due at 4pm Friday, September 16, 2005

1. **Cross Products**

   a) Starting from $(\vec{a} \times \vec{b})_k = \epsilon_{ijk} a_i b_j$ expressed in Cartesian coordinates, determine all nonzero values of ϵ_{ijk} in three dimensions.

   b) Show that $\epsilon_{ijk}\epsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$. (Hint: examine individual cases for the indices i, j, l, m.)

   c) Use this result and index notation to show $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$.

2. **Spherical Polar Basis Vectors**

Derive $\partial_a \vec{e}_b$ in spherical coordinates, where a and b are chosen from $\{r, \theta, \phi\}$. Your results should be expressed using the spherical basis vectors. In other words, find all of the coefficients ω_{abc} such that $\partial_a \vec{e}_b = \omega_{abc}\vec{e}_c$. (Hint: write the spherical polar basis vectors as linear combinations of Cartesian basis vectors.) Show that $\omega_{abc} = -\omega_{acb}$ for any orthonormal basis.

3. **Divergence and Curl in Spherical Coordinates**

   a) Using the results of Problem 2, evaluate $\vec{\nabla} \cdot \vec{e}_a$ for $a \in \{r, \theta, \phi\}$.

   b) Using the results of Problem 2, evaluate $\vec{\nabla} \times \vec{e}_a$ for $a \in \{r, \theta, \phi\}$.

   c) Derive the expressions for divergence and curl in spherical coordinates given by equations (1.71) and (1.72) of Griffiths.

4. **Practice with Vector Calculus**
Given the vector fields

$$\begin{aligned}\vec{u}(\vec{x}\,) &= r^{-2}(\vec{e}_r \times \vec{e}_z) \\ \vec{v}(\vec{x}\,) &= e^{ar^2}\vec{e}_r \\ \vec{w}(\vec{x}\,) &= xe^{-ar^2}\vec{e}_y\end{aligned}$$

calculate the following expressions:

a) $\vec{\nabla} \cdot \vec{u}(\vec{x}\,)$

b) $\vec{v}(\vec{x}\,) \cdot \vec{w}(\vec{x}\,)$

c) $\vec{\nabla} \times \vec{v}(\vec{x}\,)$

d) $\int \vec{u}(\vec{x}\,) \cdot \vec{w}(\vec{x}\,)\, d^3x$

5. **Griffiths** Problem 1.56 (p. 55)

6. **Griffiths** Problem 1.47 (p. 52)