A Hands on Introduction to NMR

22.920

lecture #2

The Rotating Frame, RF Pulses and the Bloch Equations

Review of Free Precession - Since the individual spins are quantized along the direction of the external magnetic field (along the z-axis), then only this component of the nuclear spin has a definite value at equilibrium, the two transverse components (x and y) are in superposition states. The bulk nuclear magnetization at equilibrium is a stationary magnetic moment aligned along the z-axis.

As we have seen, the dynamics of this bulk magnetization away from equilibrium can be broken down into two simple motions,

1) a precession about the applied magnetic field associated with the torque,

$$\frac{d}{dt}M(t) = \gamma M(t) \times B,$$

and,

2) a relaxation that carries the magnetization back to equilibrium.

Since the external field is about the z-axis, the precession is around the z-axis at a frequency of,

$$\omega_o = \gamma |B_o|,$$

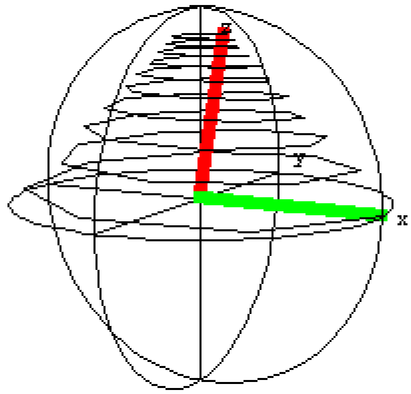
the so-called Larmor frequency.

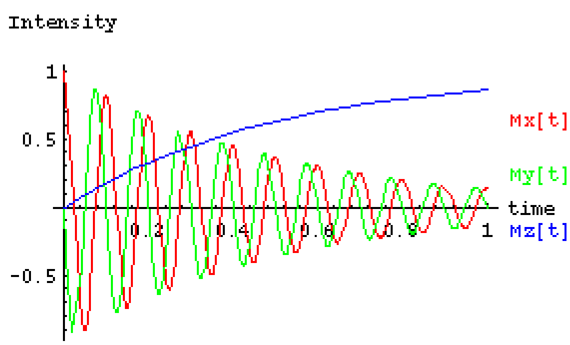
The relaxation process has two components, the magnetization along the z-axis relaxes towards its equilibrium value, M_0, and the transverse components (x and y) relax towards zero. The bulk magnetization vector is composed of the magnetic moments of many individual spins and its length has a maximum which is equal to the Boltzman excess population present at equilibrium, M_0. However, the magnetization does not have a minimum value since two individual magnetic moments may be anti-parallel to one another and thereby reduce the length of the bulk magnetization vector. For instance, if the magnet homogeniety is poor, then the FID decreased quickly due to the interference of the spin magnetization from various parts of the sample, yet the z-component of the magnetization is uneffected by this. Therefore, the transverse component can relax at a faster rate than the z component does in its return to equilibrium, and two time constants are required to describe spin relaxation. These two relaxation times are called the spin-lattice relaxation time, T_1, (along the z-axis) and the spin-spin relaxation time, T_2, (governing the decay of transverse magnetization).

The Bloch Equations - The Bloch equations describing the complete spin dynamics are,

$$\frac{d}{dt}M(t) = \gamma M(t) \times B - \frac{1}{T_1}(M_z - M_o)\hat{z} - \frac{1}{T_2}(M_x\hat{x} + M_y\hat{y}).$$

Notice that T_2 must be less than or equal to T_1 to conserve the maximum length of the magnetization vector. Two useful pictorial representations of the evolution of the spin magnetization back to equilibrium, after having first been placed along the x-axis are shown below.





These plots are a result of a simulation run with the following parameters

T1 = 0.5 sec

T2 = 0.5 sec

Δwo = 20 Hz

The simulation covers times of 0 to 1 seconds.