Note on the One Time Discount Case

There has been some confusion and questions concerning the derivation of the Q^*_g value. Let me try to clear this up with two different approaches (that are actually the same approach but they start in slightly different places). In each case I am trying to maximize the savings (the difference $TC_{NSP} - TC_{SP}$). In each case, the TC_{SP} is the same:

$$TC_{SP} = C_{pg}Q_g + C_hC_{pg}\left(\frac{Q_g}{2}\right)\left(\frac{Q_g}{D}\right) + C_o$$

That is, it is the sum of the purchase price, the order costs, and the holding costs. Note that the last term of the holding costs (Q_g/D) is the cycle time for this new quantity, Q_g, that is being ordered.

First Approach

For this approach, start with the idea that we are comparing the special quantity option against the existing optimal quantity ordered. That means that:

$$TC_{NSP} = (CycleTime)(TC^*_{NSP} + PurchaseCost)$$

$$TC_{NSP} = \left(\frac{Q_g}{D}\right)\sqrt{2C_oC_hC_pD} + \left(\frac{Q_g}{D}\right)C_pD$$

So that the savings amount becomes:

$$Savings = TC_{NSP} - TC_{SP}$$

$$= \left(\frac{Q_g}{D}\right)\sqrt{2C_oC_hC_pD} + \left(\frac{Q_g}{D}\right)C_pD - C_{pg}Q_g - C_hC_{pg}\left(\frac{Q_g}{2}\right)\left(\frac{Q_g}{D}\right) - C_o$$

Taking the first derivative and setting it equal to zero:

$$\frac{dS}{d(Q_g)} = 0 = \left(\frac{1}{D}\right)\sqrt{2C_oC_hC_pD} + \left(C_p - C_{pg}\right) - \left(\frac{2C_hC_{pg}Q_g}{2D}\right)$$

Checking for concavity – the second derivative = $-C_oC_{pg}/D < 0$, which is correct. So that

$$Q^*_g = \left(\frac{D}{DC_hC_{pg}}\right)\sqrt{2C_oC_hC_pD} + \frac{D\left(C_p - C_{pg}\right)}{C_hC_{pg}}$$

This looks like there might be an opportunity to collect terms and substitute in Q^*_w – the optimal quantity for the initial pricing. So, multiplying the first term by (C_hC_p /C_hC_p) and manipulating it, we get:

$$\boxed{Q^*_g = Q^*_w\left(\frac{C_p}{C_{pg}}\right) + \frac{D\left(C_p - C_{pg}\right)}{C_hC_{pg}}}$$

Which is the same result as in the lecture note.

Second Approach

Note that most of the terms in the TC_{NSP} equation in the lecture notes will disappear when taking the first derivative. In fact, the relevant terms are equivalent to the costs at the initial pricing level, C_p.

$$TC_{NSP} = C_p Q_g + C_h C_p \left(\frac{Q_w}{2}\right)\left(\frac{Q_g}{D}\right) + C_o\left(\frac{Q_g}{Q_w}\right)$$

The term that is giving people problems is the holding cost or middle term. In words, this is saying that my holding costs are a product of:

($/$/yr)($/unit)(Avg Inventory in units)(Cycle Time for Special Deal in years) = $

So this middle term is actually capturing the total cost of holding Q_g of inventory while still ordering Q_w each period. Check the units – it works out. So that the savings amount becomes:

$$Savings = TC_{NSP} - TC_{SP}$$
$$= C_p Q_g + C_h C_p\left(\frac{Q_w}{2}\right)\left(\frac{Q_g}{D}\right) + C_o\left(\frac{Q_g}{Q_w}\right) - C_{pg}Q_g - C_h C_{pg}\left(\frac{Q_g}{2}\right)\left(\frac{Q_g}{D}\right) - C_o$$

Taking the first derivative and setting it equal to zero:

$$\frac{dS}{d(Q_g)} = 0 = C_p + \left(\frac{C_o}{Q_w}\right) + \left(\frac{C_h C_p Q_w}{2D}\right) - C_{pg} - \left(\frac{2C_h C_{pg} Q_g}{2D}\right)$$

So that

$$Q_g^* = \frac{D\left(C_p - C_{pg}\right)}{C_h C_{pg}} + \left(\frac{DC_o}{Q_w C_h C_{pg}}\right) + \left(\frac{DC_h C_p Q_w}{2DC_h C_{pg}}\right)$$

Rearranging this:

$$Q_g^* = \frac{D\left(C_p - C_{pg}\right)}{C_h C_{pg}} + \left(\frac{2DC_o + C_h C_p Q_w^2}{2Q_w C_h C_{pg}}\right)$$

Looking here it seems like we can substitute in ($2C_oD/C_hC_p$) for the Q^2_w term. Messing around with the algebra, you should get:

$$Q_g^* = \frac{D\left(C_p - C_{pg}\right)}{C_h C_{pg}} + \left(\frac{C_p}{C_{pg}}\right)\left(\frac{2DC_o}{C_p C_h}\right)\left(\frac{1}{Q_w}\right)$$
$$Q_g^* = \frac{D\left(C_p - C_{pg}\right)}{C_h C_{pg}} + \left(\frac{C_p}{C_{pg}}\right)\left(\frac{Q_w^2}{Q_w}\right)$$

Which reduces to:

$$Q_g^* = Q_w^* \left(\frac{C_p}{C_{pg}} \right) + \frac{D\left(C_p - C_{pg}\right)}{C_h C_{pg}}$$