

Notes on Inventory Management & Control: Finite Planning Horizon (FPH) Inventory Problems

Introduction

Consider the situation where demand for an item is known in advance with certainty for a number of future periods, and the demand for the item varies from period to period. Such a situation frequently arises in material management, that is, where the item is a raw material or component part in a manufacturing process. Demand for the item for several periods in advance can be inferred from the master production schedule. How should such an item be controlled? That is,

- a. What are the relevant costs?
- b. What should the order quantities be?
- c. When should I place orders?
- d. What is the optimal ordering schedule?

We can explore these issues by modifying the basic lot-sizing problem as follows. Suppose we have:

$$C_o = \$500, C_p = \$ 50.00, C_h = 24\%, D = 2000$$

But now demands occur on the first of the month according to the following schedule:

Month	Demand
1	200
2	150
3	100
4	50
5	50
6	100
7	150
8	200
9	200
10	250
11	300
12	250

This is called a "finite planning horizon" problem because we assume demand ends after period 12. We will therefore plan our orders so that no stock remains after demand for period 12 is satisfied.

We will also make the following standard assumptions:

- a. Lead time is zero; orders arrive immediately.
- b. Demand is needed and consumed on the first day of the period. Holding costs are therefore not charged on items consumed during a period.
- c. Holding costs are charged for assets ordered in advance of need and held over for use in later periods.

For simplicity we will convert $C_h C_p$, the cost to hold one unit for one year, to C_{hp} , the cost to hold one unit for one period. Since for our data $C_h C_p = \$12.00$, and the length of the period is one month or one twelfth of a year, $C_{hp} = \$1.00$ per unit held per period held.

In general, $C_{hp} = \frac{C_h C_p}{N}$, where N = the number of planning periods in a year.

To solve this problem, we will look at five basic approaches. These approaches illustrate the pros and cons of using heuristics versus optimal approaches.

1. One Time Buy
2. Lot For Lot
3. Simple EOQ
4. Silver-Meal Algorithm
5. Optimal Solutions

Approach I: ONE TIME BUY

A simple way to approach the problem is with a one time purchase of the total requirement. This technique minimizes the total ordering cost, but accumulates large holding costs. The longer the planning horizon is, the more serious the growth in holding costs becomes. For the sample problem:

Month	Demand	Order Quantity	Holding Costs	Ordering Costs	Period Costs
1	200	2000	\$1800	\$500	\$2300
2	150	0	\$1650	0	\$1650
3	100	0	\$1550	0	\$1550
4	50	0	\$1500	0	\$1500
5	50	0	\$1450	0	\$1450
6	100	0	\$1350	0	\$1350
7	150	0	\$1200	0	\$1200
8	200	0	\$1000	0	\$1000
9	200	0	\$800	0	\$800
10	250	0	\$550	0	\$550
11	300	0	\$250	0	\$250
12	250	0	0	0	0
Totals:	2000	2000	\$13100	\$500	\$13600

Approach II: LOT FOR LOT

Another simple approach is to place an order each period for the exact amount required that period. This procedure minimizes holding costs (by forcing them to zero) but maximizes ordering costs. Length of the planning horizon has no effect, since there is no "planning ahead" in the sense of ordering inventory to store against future requirements. For the sample problem:

Month	Demand	Order Quantity	Holding Costs	Ordering Costs	Period Costs
1	200	200	0	\$500	\$500
2	150	150	0	\$500	\$500
3	100	100	0	\$500	\$500
4	50	50	0	\$500	\$500
5	50	50	0	\$500	\$500
6	100	100	0	\$500	\$500
7	150	150	0	\$500	\$500
8	200	200	0	\$500	\$500
9	200	200	0	\$500	\$500
10	250	250	0	\$500	\$500
11	300	300	0	\$500	\$500
12	250	250	0	\$500	\$500
Totals:	2000	2000	0	\$6000	\$6000

Approach III: SIMPLE EOQ

Another approach is to ignore the changed conditions and use EOQ anyway. We can calculate an average annual demand and use it to compute an optimal order quantity. Whenever available stock is insufficient to satisfy period demand, we order EOQ, or a multiple of EOQ, making sure to modify the last order to zero out inventory at the end of the planning horizon. This approach will heuristically attempt to balance ordering and holding costs, but will not necessarily produce a cost-optimal solution. In this problem $Q_w=408.2$. If we round this to 400:

Month	Demand	Order Quantity	Holding Costs	Ordering Costs	Period Costs
1	200	400	\$200	\$500	\$700
2	150	0	\$50	0	\$50
3	100	400	\$350	\$500	\$850
4	50	0	\$300	0	\$300
5	50	0	\$250	0	\$250
6	100	0	\$150	0	\$150
7	150	0	0	0	0
8	200	400	\$200	\$500	\$700
9	200	0	0	0	0
10	250	400	\$150	\$500	\$650
11	300	400	\$250	\$500	\$750
12	250	0	0	0	0
Totals:	2000	2000	\$1900	\$2500	\$4400

Note:

1. This solution is cheaper than the \$5000 total cost in the basic EOQ problem because demand is instantaneous at twelve discrete points, and so the holding costs are much easier to control.
2. As you might expect, the quality of the solution is related to the "lumpiness" of the demand schedule. If demand is relatively uniform period-to-period, the solution quality is good. If demand varies greatly from one period to the next, solution quality suffers, particularly if there are long periods of zero demand.
3. This ordering pattern generates "leftovers". The order quantity is "on average" correct, but is seldom "exactly" correct. "Leftovers" create extra holding costs.

Approach IV: THE SILVER-MEAL ALGORITHM

This is a "near optimal" heuristic which builds order quantities by taking a marginal analysis approach. We know from our analysis of the "Simple EOQ" approach that any fixed lot size will create "leftovers" which increase total cost unnecessarily. A better approach is to order "whole periods worth" of stock. But should you order one (period worth), or two, or more? As usual, it depends. At first, increasing the buy quantity saves money because order costs are reduced since fewer buys are made. Eventually, though, large order quantities will begin to increase total costs as holding costs rise.

The Silver-Meal Algorithm trades-off ordering and holding costs by analyzing the problem "one buy at a time". The idea is, should the first buy cover period 1, periods 1 and 2, periods 1, 2, and 3, and so forth. To answer this question, the procedure considers each potential buy quantity sequentially and calculates the "average cost per period covered" as the sum of the ordering and holding costs implied by the potential buy divided by the number of periods which would be covered by such an order. Simply put, the decision rule is : "Add the next period's demand to the current order quantity unless the average cost per period covered would not be reduced." That is, as long as the average cost per period covered by the order would be reduced by adding an additional period worth to the order, we will do so. If adding an additional period worth to the order would not reduce the average cost per period covered, then we will consider that the order size is determined, and we will begin to calculate the next order using the same procedure. We will continue one order at a time until every period has been covered with an order.

Procedure:

Mon	Dmd	Lot QTY	Order Cost	Holding Costs	Lot Cost	Mean Cost
FIRST BUY:						
1	200	200	\$500	\$0	\$500	\$500
2	150	350	\$500	\$150	\$650	\$325
3	100	450	\$500	$\$150 + \$200 = \$350$	\$850	\$283
4	50	500	\$500	$\$150 + \$200 + \$150 = \500	\$1000	\$250
5	50	550	\$500	$\$150 + \$200 + \$150 + \$200 = \$700$	\$1200	\$240
6	100	650	\$500	$\$150 + \$200 + \$150 + \$200 + \$500 = \1200	\$1700	\$283
SECOND BUY:						
6	100	100	\$500	\$0	\$500	\$500
7	150	250	\$500	\$150	\$650	\$325
8	200	450	\$500	$\$150 + \$400 = \$550$	\$1050	\$350
THIRD BUY:						
8	200	200	\$500	\$0	\$500	\$500
9	200	400	\$500	\$200	\$700	\$350
10	250	650	\$500	$\$200 + \$500 = \$700$	\$1200	\$400
FOURTH BUY:						
10	250	250	\$500	\$0	\$500	\$500
11	300	550	\$500	\$300	\$800	\$400
12	250	800	\$500	$\$300 + \$500 = \$800$	\$1300	\$433
FIFTH BUY:						
12	250	250	\$500	\$0	\$500	\$500

SOLUTION:

Month	Demand	Order Quantity	Holding Costs	Ordering Costs	Period Costs
1	200	550	\$350	\$500	\$850
2	150	0	\$200	0	\$200
3	100	0	\$100	0	\$100
4	50	0	\$50	0	\$50
5	50	0	0	0	0
6	100	250	\$150	\$500	\$650
7	150	0	0	0	0
8	200	400	\$200	\$500	\$700
9	200	0	0	0	0
10	250	550	\$300	\$500	\$800
11	300	0	0	0	0
12	250	250	0	\$500	\$500
Totals:	2000	2000	\$1350	\$2500	\$3850

Heading V: OPTIMAL PROCEDURES

A: WAGNER WHITIN ALGORITHM.

WW is a considerably more laborious procedure than Silver-Meal which is based on the principles of dynamic programming.

B: INTEGER PROGRAMMING.

This problem can be formulated and solved exactly as an integer program. Zero-one integer variables are introduced to accommodate the ordering costs.

A COMPARISON OF TECHNIQUES

Month	Demand	OTB	LFL	"EOQ"	S-M	OPT
1	200	2000	200	400	550	550
2	150		150			
3	100		100	400		
4	50		50			
5	50		50			
6	100		100		250	450
7	150		150			
8	200		200	400	400	
9	200		200			450
10	250		250	400	550	
11	300		300	400		550
12	250		250		250	
TOTAL COST:		\$13,600	\$6,000	\$4,400	\$3,850	\$3,750

A COMMENT ON OPTIMALITY

Optimal solutions trade-off ordering and holding costs across time periods based on the certainty of the demand schedule. In practice, the procedure would be re-run each month, with a new month added on the end, and the old month eliminated. Only the most immediate orders would be placed; the later orders would be held. In this sort of "rolling horizon" application, short-term look-ahead

procedures like Silver-Meal typically can out-perform the "optimal" approaches, particularly if updates are made to demand forecasts within the planning horizon.