

Learning 5

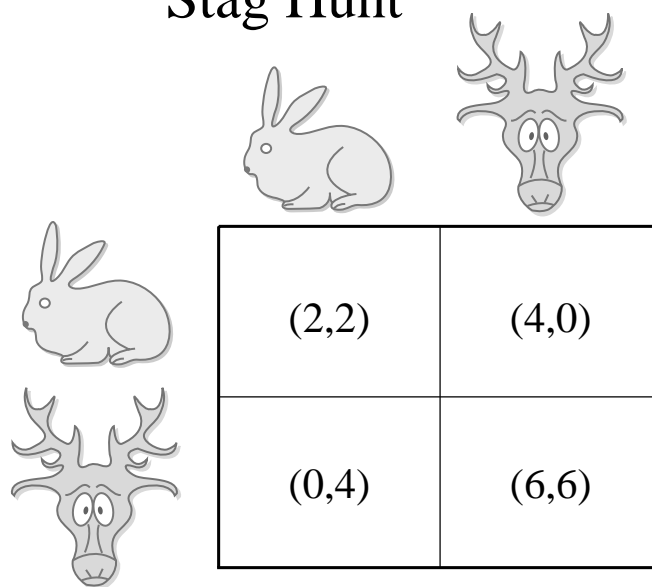
Adjustment with persistent noise (Kandori, Mailath, Rob)

14.126 Game Theory

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Stag Hunt



A 2x2 payoff matrix for the Stag Hunt game. The columns represent the strategies 'Rabbit' and 'Stag', and the rows represent the strategies 'Rabbit' and 'Stag'. The payoffs are (2,2) for (Rabbit, Rabbit), (4,0) for (Rabbit, Stag), (0,4) for (Stag, Rabbit), and (6,6) for (Stag, Stag). The icons for a rabbit and a stag head are placed around the matrix to indicate the strategies.


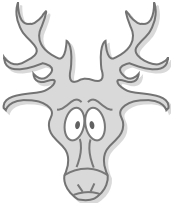

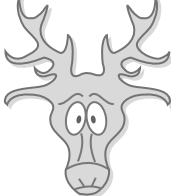
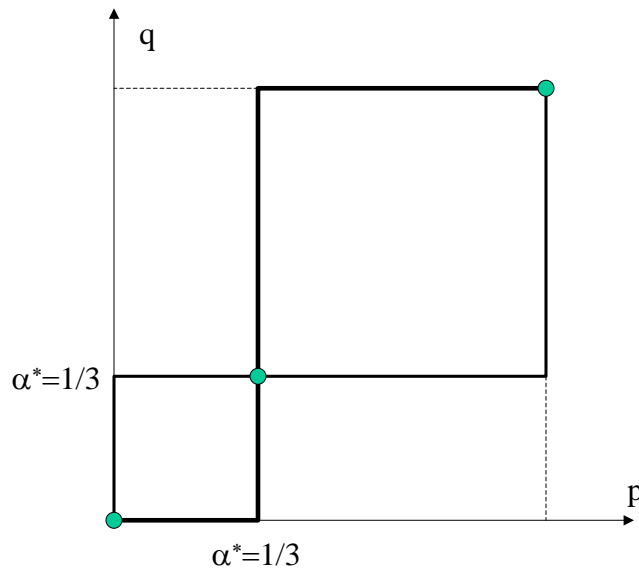
		
	(2,2)	(4,0)
	(0,4)	(6,6)

Figure by MIT OCW.

Best responses in Stag-Hunt game



Adjustment Process

- N = population size.
- State space: θ_t = # of players who play A at t .
- $u_A(\theta_t) = \theta_t/N u(A,A) + (N - \theta_t)/N u(A,B)$
- $\theta_{t+1} = P(\theta_t)$, where

$$P(\theta_t) > \theta_t \Leftrightarrow u_A(\theta_t) > u_B(\theta_t) \ \&$$

$$P(\theta_t) = \theta_t \Leftrightarrow u_A(\theta_t) = u_B(\theta_t).$$

- Example:

$$P(\theta_t) = BR(\theta_t) = \begin{cases} N & \text{if } u_A(\theta_t) > u_B(\theta_t) \\ \theta_t & \text{if } u_A(\theta_t) = u_B(\theta_t) \\ 0 & \text{if } u_A(\theta_t) < u_B(\theta_t) \end{cases}$$

Noise

- Independently, each agent with probability 2ε mutates, and plays either of the strategies with equal probabilities.

$$P^\varepsilon = \begin{bmatrix} (1-\varepsilon)^N & (1-\varepsilon)^N & \dots & (1-\varepsilon)^N & \varepsilon^N & \dots & \varepsilon^N \\ N(1-\varepsilon)^{N-1}\varepsilon & N(1-\varepsilon)^{N-1}\varepsilon & \dots & N(1-\varepsilon)^{N-1}\varepsilon & N(1-\varepsilon)\varepsilon^{N-1} & \dots & N(1-\varepsilon)\varepsilon^{N-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ N(1-\varepsilon)\varepsilon^{N-1} & N(1-\varepsilon)\varepsilon^{N-1} & \dots & N(1-\varepsilon)\varepsilon^{N-1} & N(1-\varepsilon)^{N-1}\varepsilon & \dots & N(1-\varepsilon)^{N-1}\varepsilon \\ \varepsilon^N & \varepsilon^N & \dots & \varepsilon^N & (1-\varepsilon)^N & \dots & (1-\varepsilon)^N \end{bmatrix}$$

- $\varphi^*(\varepsilon) =$ invariant distribution for P^ε .

P^ε

$(1-\varepsilon)^N$ $M(1-\varepsilon)^{N-1}\varepsilon$ \vdots $\binom{N}{N^*-1}(1-\varepsilon)^{N-N^*+1}\varepsilon^{N^*-1}$ <hr style="border: 0.5px solid black;"/> $\binom{N}{N^*}(1-\varepsilon)^{N-N^*}\varepsilon^{N^*}$ \vdots ε^N	\dots \dots \ddots \dots \dots \dots \dots	$(1-\varepsilon)^N$ $M(1-\varepsilon)^{N-1}\varepsilon$ \vdots $\binom{N}{N^*-1}(1-\varepsilon)^{N-N^*+1}\varepsilon^{N^*-1}$ <hr style="border: 0.5px solid black;"/> $\binom{N}{N^*}(1-\varepsilon)^{N-N^*}\varepsilon^{N^*}$ \vdots ε^N	\dots \dots \ddots \dots \dots \dots \dots	ε^N $N(1-\varepsilon)\varepsilon^{N-1}$ \vdots $\binom{N}{N^*-1}(1-\varepsilon)^{N^*-1}\varepsilon^{N-N^*+1}$ <hr style="border: 0.5px solid black;"/> $\binom{N}{N^*}(1-\varepsilon)^{N^*}\varepsilon^{N-N^*}$ \vdots $(1-\varepsilon)^N$	\dots \dots \ddots \dots \dots \dots \dots	ε^N $M(1-\varepsilon)\varepsilon^{N-1}$ \vdots $\binom{N}{N^*-1}(1-\varepsilon)^{N^*-1}\varepsilon^{N-N^*+1}$ <hr style="border: 0.5px solid black;"/> $\binom{N}{N^*}(1-\varepsilon)^{N^*}\varepsilon^{N-N^*}$ \vdots $(1-\varepsilon)^N$
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Invariant Distribution

- $N^* = \lceil \alpha^* N \rceil < N/2$.
- $D_A = \{\theta | \theta \geq N^*\}$; $D_B = \{\theta | \theta < N^*\}$;
- $q_{AB} = \Pr(\theta_{t+1} \in D_A | \theta_t \in D_B)$;
- $q_{BA} = \Pr(\theta_{t+1} \in D_B | \theta_t \in D_A)$;
- $$p_A(\varepsilon) = \sum_{\theta \in D_A} \varphi_\varepsilon^*(\theta)$$
- $p_B(\varepsilon) = 1 - p_A(\varepsilon)$.

Invariant distribution, continued

- $$\begin{bmatrix} p_A(\varepsilon) \\ p_B(\varepsilon) \end{bmatrix} = \begin{bmatrix} 1 - q_{BA} & q_{AB} \\ q_{BA} & 1 - q_{AB} \end{bmatrix} \begin{bmatrix} p_A(\varepsilon) \\ p_B(\varepsilon) \end{bmatrix}$$
- $$\begin{bmatrix} -q_{BA} & q_{AB} \\ q_{BA} & -q_{AB} \end{bmatrix} \begin{bmatrix} p_A(\varepsilon) \\ p_B(\varepsilon) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
- $$\frac{p_A(\varepsilon)}{p_B(\varepsilon)} = \frac{q_{AB}}{q_{BA}}$$

Invariant distribution, continued

$$\begin{aligned}
 \frac{p_A(\varepsilon)}{p_B(\varepsilon)} &= \frac{q_{AB}}{q_{BA}} \\
 &= \frac{\binom{N}{N^*} \varepsilon^{N^*} (1-\varepsilon)^{N-N^*} + \binom{N}{N^*+1} \varepsilon^{N^*+1} (1-\varepsilon)^{N-N^*-1} + \dots}{\binom{N}{N^*} \varepsilon^{N-N^*} (1-\varepsilon)^{N^*} + \binom{N}{N^*-1} \varepsilon^{N-N^*+1} (1-\varepsilon)^{N^*-1} + \dots} \\
 &= \frac{\binom{N}{N^*} \varepsilon^{N^*} (1-\varepsilon)^{N-N^*} + o(\varepsilon^{N^*})}{\binom{N}{N^*} \varepsilon^{N-N^*} (1-\varepsilon)^{N^*} + o(\varepsilon^{N-N^*})} \cong \frac{1}{\varepsilon^{N-2N^*}} \rightarrow \infty.
 \end{aligned}$$

Proposition

If N is large enough so that $N^* < N/2$, then limit φ^* of invariant distributions puts a point mass on $\theta = N$, corresponding to all players playing A.