Interactive Epistemology –II

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Road Map

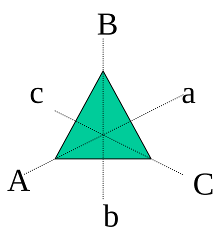
1. Summary
2. An Example
3. Agreement and No-trade theorems
4. Universal State Space
5. A state space for games
6. Epistemic Foundations for Correlated Equilibrium and Rationalizability

Summary

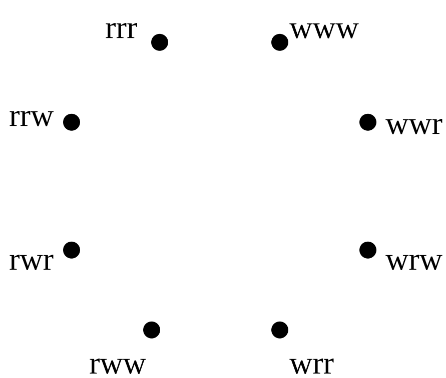
$\kappa(\omega)$	
$I(\omega) = \{\omega' \in \Omega \mid \kappa(\omega)=\kappa(\omega')\}$	$I_{1,\ldots,n}(\omega) = \bigcap_{\omega \in F \in \mathfrak{K}_1 \cap \ldots \cap \mathfrak{K}_n} F$
$\mathfrak{I} = \{I(\omega) \mid \omega \in \Omega\}$	$\mathfrak{I}_1 \wedge \mathfrak{I}_2 \wedge \ldots \wedge \mathfrak{I}_n$
$\mathfrak{K} = \{KE \mid E \in \mathfrak{E}\}$ $\mathfrak{K} = \{\bigcup_{\omega \in \Omega'} I(\omega) \mid \Omega' \subseteq \Omega\}$	$\mathfrak{K}_1 \cap \mathfrak{K}_2 \cap \ldots \cap \mathfrak{K}_n$
$KE = \{\omega \in \Omega \mid I(\omega) \subseteq E\}$	$CKE_{1,\ldots,n}$

Example

$$CK_{AB}E \cap CK_{BC}E \cap CK_{AC}E \neq CK_{ABC}E$$



Red or White?



Agreement Theorem

- B = a finite set of decisions b;
- d : E \ {∅} → B, a decision rule;
- d satisfies the sure-thing principle: if $E = \cup_a J_a$ where $\{J_a\}$ is a family of disjoint sets, and if $d(J_a) = b$ at each a, then d(E) = b.

Theorem: For each i, define $d_i \Omega \to B$ by $d_i(\omega)=d(I_i(\omega))$. Then,

$$CK(d_1=b) \cap CK(d_2=c) \neq \emptyset \Rightarrow b = c.$$

Proof: Assume: $\omega \in E := CK(d_1=b) \cap CK(d_2=c)$.

1. $\exists \{J_a\} \subseteq \mathfrak{I}_1$ s.t. $E = \cup_a J_a$.
2. $d(J_a) = b$ for each a.
3. d(E) = b.

Application – agreeing to disagree

- A = a fixed event
- B = [0,1]
- d(E) = P(A|E), conditional probability w.r.t. a common prior P given E.
- Bayes' rule => d satisfies the sure-thing principle.
- Agreement Theorem => if it is common knowledge at w that $P(A|I_1(\omega)) = b$ and $P(A|I_2(\omega)) = b$, then b = c.

No-trade Theorem

- $\omega = (x,z)$; $z = (z_1,\ldots,z_n)$, i observes z_i, owns $e_i(x)$.
- $y : \Omega \to B$, $y \in Y$.
- $u_i(y_i(\omega);x) := \underline{u}_i(y_i(\omega) + e_i(x);x)$.

Lemma: Assume that y is Pareto-optimal, and y' and $A \subseteq \Omega$ are s.t. $E[u_i(y_i'(\omega);x)|A] \geq E[u_i(y_i(\omega);x)|A]$ and $Prob(A) > 0$. Then, $E[u_i(y_i'(\omega);x)|A] = E[u_i(y_i(\omega);x)|A]$. If each u_i is strictly concave, then y' = y on A.

Proof: Define y* = [y' on A; y on ~A]. Apply the sure-thing principle. Then, $E[u_i(y_i^*(\omega);x)] \geq E[u_i(y_i(\omega);x)]$. If $E[u_i(y_i'(\omega);x)|A] > E[u_i(y_i(\omega);x)|A]$, then $E[u_i(y_i^*(\omega);x)] > E[u_i(y_i(\omega);x)]$.

No-trade Theorem

Theorem: Assume that $\underline{y} = 0$ is Pareto-optimal, and $Prob(I_{1,\ldots,n}(\omega)) > 0$. If it is common knowledge at ω that y is feasible and each i weakly prefers y to $\underline{y}$, then each is indifferent between y and $\underline{y}$. If each agent is strictly risk averse, then $y = \underline{y}$.

<u>Proof:</u> Take $A = I_{1,\ldots,n}(\omega)$ in the lemma.

Universal state-space

- X = an alphabet of letters x, y, z, …
- Formulas: finite strings of symbols s.t.
  - Every letter is a formula;
  - If f and g are formulas, so is (f)OR(g);
  - If f is a formula, so are NOT(f) and k_if.
- A *list* L (of formulas) is *logically closed* iff

  [f∈L & (f => g)∈L] => g∈L;
- *Epistemically closed* iff

  f ∈ L => k_if ∈ L.

- A state ω is any logically closed list s.t.
  - f∈ω ⇔ NOT(f)∉ω;
  - ω includes all the "tautologies."
- Ω = the set of all states.
- κ_i(w) = {k_i(f)| k_i(f) ∈ω}

Tautologies

The set of tautologies is the smallest logically and epistemically closed list that contains all:

- (f OR f) => f
- f => (f OR g)
- (f OR g) => (g OR f)
- (f => g) =>

  ((h OR f) => (h OR g))
- k_i(f) => f
- (k_i(f=>g)) => (k_i(f)=>k_i(g))
- k_i(f) => k_i(k_i(f))
- NOT(k_i(f)) => k_i(NOT(k_i(f)))

Some theorems

- $E_f := \{\omega \in \Omega | f \in \omega\}$
- $\sim E_f = E_{NOT(f)}$
- $E_f \cup E_g = E_{(f\ OR\ g)}$
- $E_f \cap E_g = E_{(f\ \&\ g)}$
- $K_i\ E_f = E_{ki(f)}$
- $E_f \subseteq E_g \Leftrightarrow$ [f => g is a tautology]