

**Problem Set 5**  
**(due on the day of Lecture # 7)**

**Problem 1** (Unanticipated and anticipated changes in government expenditures)

Consider the decentralized Ramsey economy with government expenditures of problem 2 part 5 in problem set 4. Recall that the dynamic system associated with this economy is given by

$$\begin{aligned}\dot{k}_t &= f(k_t) - c_t - g - nk_t, \\ \frac{\dot{c}_t}{c_t} &= \sigma(c_t)(f'(k_t) - \rho - n),\end{aligned}$$

where the initial per capita capital stock  $k_0$  is given and the paths of consumption and capital satisfy the transversality condition  $\lim_{t \rightarrow \infty} k_t u'(c_t) e^{-\rho t} = 0$ . Here  $g > 0$  is the fixed level of per capita government expenditure per unit of time. Assume that at time  $t = 0$  the economy is in steady state, i.e.  $k_0$  satisfies  $f'(k_0) = \rho + n$ , and that it has been in steady state for some time. Suppose that at time  $t = 0$  the government permanently reduces government expenditures from  $g$  to  $g_L$  where  $0 \leq g_L < g$ . Moreover, suppose that this change is unanticipated (if it were anticipated, families would adjust their behaviour already before time  $t = 0$  and it may not be reasonable to assume that the economy is in steady state at time  $t = 0$ ).

1. Draw the phase diagram corresponding to the old level of government expenditures  $g$ , including the saddle path. How does the reduction in government expenditures to  $g_L$  change the phase diagram and the steady state? Indicate in the phase diagram how the economy adjusts to the unanticipated reduction in government expenditures. Sketch the time paths of per capita capital, consumption and government expenditures (starting at some time  $t < 0$ ).
2. Now suppose that the reduction in government expenditures at time  $t = 0$  continues to be unanticipated but is known to be temporary, i.e. families anticipate a return to the old level of government expenditures  $g$  to occur at some time  $\bar{t} > 0$ . Is it possible for consumption to jump at time  $t = \bar{t}$ ? Repeat the analysis from part 1. How do the answers depend on the duration of the reduction in government expenditures?
3. Next suppose that at time  $t = 0$  the government announces that at time  $\underline{t} > 0$  government expenditures will be permanently reduced to  $g_L$ . So between times  $t = 0$

and  $t = \underline{t}$  the old level  $g$  still applies, and the reduction to  $g_L$  at time  $\underline{t}$  is now anticipated (but the announcement at time  $t = 0$  is unanticipated). Repeat the analysis from part 1.

4. Finally suppose that at time  $t = 0$  the government makes an unanticipated announcement that government expenditures will be reduced to  $g_L$  between time  $\underline{t} > 0$  and  $\bar{t} > \underline{t}$ , i.e. now the reduction is both anticipated and temporary. Repeat the analysis from part 1.

**Problem 2** (The stability of fiscal policy)

Let  $G(t)$  and  $T(t)$  denote government's real purchases and tax revenue at time  $t$ . Then the flow budget constraint of the government is given by

$$\dot{D}(t) = r(t)D(t) + G(t) - T(t)$$

where  $D(t)$  denotes the amount of debt outstanding at time  $t$ . The rate of change in debt outstanding  $\dot{D}(t)$  is the budget deficit, while  $G(t) - T(t)$  is referred to as the primary deficit. Define  $d(t)$  to be the ratio of debt to output  $d(t) = \frac{D(t)}{Y(t)}$ . Assume that  $Y(t)$  grows at a constant rate  $g > 0$ .

Assume that the interest rate is an increasing function of the debt-to-output ratio:  $r(t) = r(d(t))$ , where  $r'(\cdot) > 0$ ,  $r''(\cdot) > 0$ ,  $\lim_{d \rightarrow -\infty} r(d) < g$  and  $\lim_{d \rightarrow \infty} r(d) > g$ .

1. Suppose the budget deficit to output ratio is constant:  $\frac{\dot{D}(t)}{Y(t)} = a$ , where  $a > 0$ . Find an expression for  $\dot{d}(t)$  in terms of  $a$ ,  $g$  and  $d(t)$ . Sketch  $\dot{d}(t)$  as a function of  $d(t)$ . Is this system stable?
2. Suppose that the ratio of the primary deficit to output is constant and equal to  $a > 0$ . Find an expression for  $\dot{d}(t)$  in terms of  $a$ ,  $g$  and  $d(t)$ . Sketch  $\dot{d}(t)$  as a function of  $d(t)$ . In the case where  $a$  is sufficiently small that  $\dot{d}$  is negative for some values of  $d$ , what are the stability properties of the system? What about the case where  $a$  is sufficiently large that  $\dot{d}$  is positive for all values of  $d$ ?