(d) Suppose output is given by $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$, $0 < \alpha < 1$. Solve for w_t, the wage rate (assume labor is paid its marginal product). Now suppose there is a positive technology shock at time 1, $A_1 = A$, $A > 1$ (assume that at time 2, technology returns to $A_2 = 1$). What is the effect, if any, on the relative wage and on the relative demand for leisure? Does it make sense?

(e) How does the relative demand for leisure depend on the interest rate?, on the time preference rate?

(f) Explain intuitively why γ affects the responsiveness of labor supply to wages and the interest rate.

(g) Solve for the Euler equation, that is express the relationship between c_1 and c_2. What if $\rho = r$?

(h) Now assume that the household has initial wealth of amount $Z > 0$. Does the Euler equation derived in part g continue to hold?

Problem 3 Romer Problem 4.8 (A simplified RBC model with additive technology shocks). Consider an economy consisting of a constant population of infinitely-lived individuals. The representative individual maximizes the expected value of

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t}\, u(C_t), \quad \rho > 0$$
$$\text{where } u(C_t) = C_t - \theta C_t^2,\ \theta > 0$$

Assume that C is always in the range where $u\prime(C)$ is positive.
Output is linear in capital, plus an additive disturbance: $Y_t = AK_t + e_t$. There is no depreciation; thus $K_{t+1} = K_t + Y_t - C_t$, and the interest rate is A. Assume $A \equiv r = \rho$. Finally, the disturbance follows a first-order autoregressive process: $e_t = \phi e_{t-1} + \varepsilon_t$, where $-1 < \phi < 1$ and where the ε's are mean zero, i.i.d shocks.

1. Find the first-order condition (Euler equation) relating C_t and expectations of C_{t+1}.(Hint: set up the Bellman equation and maximize w.r.t K_{t+1} after substituting for C_t as functions of K_t, K_{t+1} etc.)

2. Guess that consumption takes the form $C_t = \alpha + \beta K_t + \gamma e_t$. Given this guess, what is K_{t+1} as a function of K_t and e_t?

3. What values must the parameters α, β, and γ have for the first-order condition in part 1 to be satisifed for all values of K_t and e_t?

4. What are the effects of a one-time shock to ε (suppose $\Delta\varepsilon_t = 1$) on the paths of Y, K, and C ?