

Answer keys for problem set 4

Exercise 8.10

Start by taking the full derivative of Dave's demand, then use the Slutsky equation to get:

$$\begin{aligned} \frac{dx_i}{dp_j}(p, p\bar{x}) &= \frac{\partial x_i}{\partial p_j}(p, p\bar{x}) + \frac{\partial x_i}{\partial m}\bar{x}_j \\ &= \frac{\partial h_i}{\partial p_j}(p, p\bar{x}) + \frac{\partial x_i}{\partial m}(\bar{x}_j - x_j) \end{aligned}$$

If Dave's optimal choice is such that $x_1 < \bar{x}_1$, then $x_2 > \bar{x}_2$. If p_2 goes down then the previous optimal bundle is still affordable so Dave is better off. If p_1 goes down, then Dave cannot afford the previous bundle anymore. He can be either better off or worse off. See the figures.

The rate of return on the consumption good is $\frac{p_1}{p_2} - 1$.

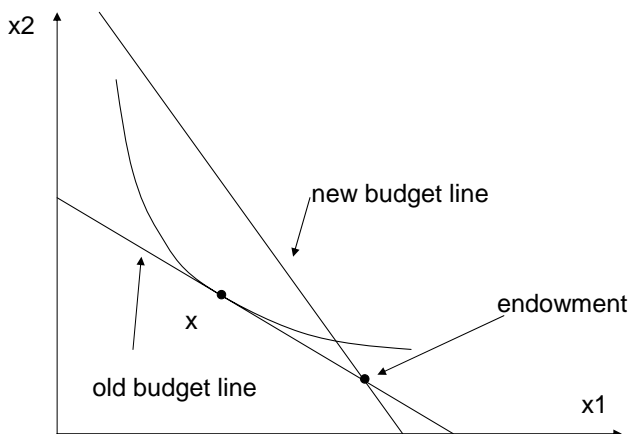


Figure 1: p_2 goes down, Dave is better off.

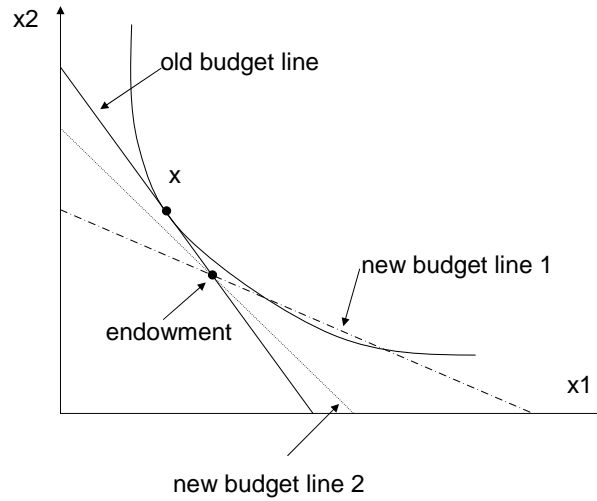


Figure 2: p_1 goes down, Dave can be either better off or worse off.

Exercise 8.12

$$v(p, y) = f(p)y$$

$$e(p, u) = \frac{u}{f(p)}$$

$$\mu(p; q, y) = e(p, v(q, y)) = \frac{f(q)}{f(p)}y$$

Exercise 8.14

The general maximization problem is:

$$\max_{S_1, S_2, C} S_1^\alpha S_2^\beta C^\gamma$$

$$C + S_1 + S_2 = Y \quad (\lambda)$$

$$S_1 \leq L \quad (\mu)$$

$$L = S_1^\alpha S_2^\beta C^\gamma - \lambda(C + S_1 + S_2 - Y) - \mu(S_1 - Y)$$

- If the L constraint is not binding, then $\mu = 0$

$$S_1 = \frac{\alpha}{\alpha + \beta + \gamma} Y < L$$

$$S_2 = \frac{\beta}{\alpha + \beta + \gamma} Y$$

$$C = \frac{\gamma}{\alpha + \beta + \gamma} Y$$

- If the L constraint is binding, then $S_1 = L$

$$\begin{aligned} S_1 &= L \\ S_2 &= \frac{\beta}{\beta + \gamma} (Y - L) \\ C &= \frac{\gamma}{\beta + \gamma} (Y - L) \end{aligned}$$

Exercise 8.16

- True. Good 1 is a normal good then demand for good 1 goes up $x_1^*(m + g_1) \geq x_1^*(m) \geq g_1$. Then the unconstrained consumption is the same as the constrained one.
- False. Good 1 is an inferior good for all income above the original m , then demand for good 1 goes down $x_1^*(m + g_1) < x_1^*(m)$. It is possible to find cases where it goes enough down so that $x_1^*(m + g_1) < g_1$. Then the unconstrained consumption is not the same as the constrained one. The consumer will choose $x_1 = g_1$ and $x_2 = m$ if the constraint is binding.
- Homothetic preferences means that income expansion paths are rays through the origin. Then the consumer would like to consume three times more good 2 than good 1 as long as the prices are unchanged, i.e. spend one fourth of his income on good 1. If the consumer is constrained to buy at least g_1 units of good 1, he will consume $\max(g_1, \frac{m+g_1}{4})$. As $m = 48$, the kink will be at $g_1 = 16$. Note that in the question, I've dropped the constraint $g_1 \leq x_1^*$ in order to have a kink in the demand function.

Exercise 9.2

When the demand function is given, the indirect money metric function can be found by integrating the following differential equation:

$$\begin{aligned} \frac{\partial \mu}{\partial p}(p; q, m) &= x(p, m) = a + bp \\ \mu(q; q, m) &= m \end{aligned}$$

The indirect money metric function is then:

$$\mu(p; q, m) = ap + \frac{b}{2}p^2 + m - (aq + \frac{b}{2}q^2)$$

By definition $\mu(p; q, m) = e(p, v(q, m))$, you can conclude that:

$$\begin{aligned}
v(q, m) &= m - (aq + \frac{b}{2}q^2) \\
e(p, u) &= ap + \frac{b}{2}p^2 + u
\end{aligned}$$

Then to get the direct utility function you can solve:

$$\begin{aligned}
u(x, z) &= \min_p v(p, m) \\
&\quad px + z = m \\
&= \min_p px + z - (ap + \frac{b}{2}p^2)
\end{aligned}$$

which leads to

$$u(x, z) = z + \frac{(x - a)^2}{2b}$$

Note: this exercise introduces the demand for a single good x . If this was the only good the consumer could buy, using the non satiation property he would spend all his income on it and his demand would be $\frac{m}{p}$. Here his demand is $x = a + bp$, therefore the consumer can choose between this good and something else. You can then assume that this is a two-good case and call the other good z with a price normalized to 1 (it is always possible to fix one price as changing all the prices by the same multiplicative factor doesn't change anything). Or you can think as the consumer deciding first how to allocate his budget between good x and the rest.

Exercise 9.4

The demand functions cannot be any set of functions of p . Those functions are derived from an optimization problem and therefore the substitution matrix has to be symmetric and negative semidefinite.

$$\left(\begin{array}{cc} \frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial m}x_1 & \frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial m}x_1 \\ \frac{\partial x_1}{\partial p_2} + \frac{\partial x_1}{\partial m}x_2 & \frac{\partial x_2}{\partial p_2} + \frac{\partial x_2}{\partial m}x_2 \end{array} \right) = \left(\begin{array}{cc} b_{11} & b_{12} \\ b_{21} & b_{22} \end{array} \right)$$

Then $b_{12} = b_{21}$, $b_{11} \leq 0$, and $b_{11}b_{22} - b_{12}^2 \geq 0$.

Using the same approach as in the previous exercise, the indirect money metric function is solution of:

$$\begin{aligned}
\frac{\partial \mu}{\partial p_1}(p; q, m) &= a_1 + b_{11}p_1 + b_{12}p_2 \\
\frac{\partial \mu}{\partial p_2}(p; q, m) &= a_2 + b_{12}p_1 + b_{22}p_2 \\
\mu(q; q, m) &= m
\end{aligned}$$

You can rewrite those equations using matrix notations:

$$\begin{aligned}D\mu(p; q, m) &= A + Bp \\ \mu(q; q, m) &= m\end{aligned}$$

with $A = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $B = \begin{pmatrix} b_1 & b_{12} \\ b_{12} & b_2 \end{pmatrix}$ and $p = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$.

The solution is:

$$\begin{aligned}\mu(p; q, m) &= p^t A + \frac{1}{2} p^t B p + m - q^t A - \frac{1}{2} q^t B q \\ &= a_1 p_1 + a_2 p_2 + \frac{1}{2} (b_1 p_1^2 + 2b_{12} p_1 p_2 + b_2 p_2^2) + m \\ &\quad - a_1 q_1 - a_2 q_2 - \frac{1}{2} (b_1 q_1^2 + 2b_{12} q_1 q_2 + b_2 q_2^2)\end{aligned}$$