MIT 14.04 Intermediate Microeconomic Theory

Fall 2003

Answer keys for problem set 6

Exercise 7.2

Call X^1 and X^2 the total amounts of good 1 and 2. Consider two consumers with the same Leontief utility function

$$u_i(x_i) = \min(\frac{x_i^1}{X^1}, \frac{x_i^2}{X^2})$$

For any endowment (ω_1, ω_2), any price p (different from $(0,0)$) is part of a Walrasian equilibrium. Indeed consider the allocation

$$x_i^j = X^j \frac{p.\omega_i}{p.X}$$

(p, x) is a Walrasian equilibrium.

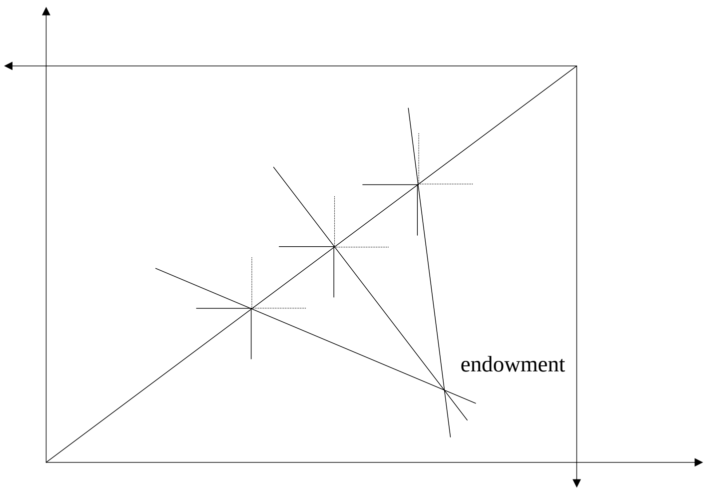

Figure 1: Infinite number of prices that are part of a Walrasian equilibrium.

Exercise 7.4

Consumer A has a Cobb-Douglas utility function, his demand functions are

$$\begin{aligned} x_A^1 &= a\frac{m_A}{p_1} = a\frac{p_2}{p_1} \\ x_A^2 &= (1-a)\frac{m_A}{p_2} = 1-a \end{aligned}$$

The demand functions of consumer B are

$$x_B^1 = x_B^2 = \frac{m_B}{p_1 + p_2} = \frac{p_1}{p_1 + p_2}$$

By Walras's law, if one market is cleared the other one will also be cleared. Check for example that the market for good 2 has to clear:

$$x_A^2 + x_B^2 = 1 \iff \frac{p_1}{p_2} = \frac{a}{1-a}$$

In equilibrium the prices will be such that $\frac{p_1}{p_2} = \frac{a}{1-a}$ and the allocation will be $x_A^1 = x_A^2 = 1 - a$ and $x_B^1 = x_B^2 = a$.

Exercise 7.6

Those agents have Cobb-Douglas utility functions

$$\begin{aligned} u_1 &= x_1^a x_2^{1-a} \\ u_2 &= x_1^b x_2^{1-b} \end{aligned}$$

Hence their demand functions are

$$\begin{aligned} x_1^1 &= a\frac{p_1 + p_2}{p_1} \\ x_1^2 &= (1-a)\frac{p_1 + p_2}{p_2} \\ x_2^1 &= b\frac{p_1 + p_2}{p_1} \\ x_2^2 &= (1-b)\frac{p_1 + p_2}{p_2} \end{aligned}$$

The market for good 1 has to clear

$$x_1^1 + x_2^1 = 2 \iff \frac{p_1}{p_2} = \frac{a+b}{2-a-b}$$

Exercise 7.8

Let an allocation be defined by (x_A^1, x_A^2) where A gets (x_A^1, x_A^2) and B gets $(1 - x_A^1, 2 - x_A^2)$.

The set of strongly Pareto efficient allocations is $S = \{(1,0), (0,2)\}$.

The set of weakly Pareto efficient allocations is $W = \{(x_A^1, 0) \text{ s.t. } x_A^1 \in [0,1]\} \cup \{(x_A^1, 2) \text{ s.t. } x_A^1 \in [0,1]\} \cup \{(x_A^1, 1) \text{ s.t. } x_A^1 \in [0,1]\} \cup \{(1, x_A^2) \text{ s.t. } x_A^2 \in [0,1]\} \cup \{(0, x_A^2) \text{ s.t. } x_A^2 \in [1,2]\}$

Exercise 7.10

The demonstration is identically the same as the one on p321, until the equality

$$\left[\sum_{i=1}^{k} \max(0, z_i(p^*))\right] p^*.z(p^*) = \sum_{i=1}^{k} z_i(p^*) \max(0, z_i(p^*))$$

The LHS is non-positive as $max(0, z_i(p^*) \geq 0$ and $p^*.z(p^*) \leq 0$. The RHS is non-negative as $z_i(p^*) \max(0, z_i(p^*))$ is either 0 or $z_i(p^*)^2$. Therefore both the LHS and the RHS must be equal to 0.

$$\sum_{i=1}^{k} z_i(p^*) \max(0, z_i(p^*)) = 0$$

And the end of the proof remains the same.