11 Monopoly

Free to set price-quantity combinations.

$$\begin{aligned} py - c(y) &\to \max_{p,y}, \\ \text{s.t. } D(p) &= y. \end{aligned}$$

Alternatively

$$p(y)y - c(y) \to \max_{y}.$$

$$\begin{aligned} FOC &: p(y) + p'(y)y = c'(y), \\ SOC &: 2p'(y) + p''(y)y - c''(y^*) \leq 0. \end{aligned}$$

Level of production is not efficient!

Natural monopolies.

Residual monopolies.

11.1 Price discriminations

In general, monopoly can offer (P, Q), or even a schedule of offers (P_j, Q_j), $j \in \mathcal{J}$.

11.1.1 1st Degree

M. extracts full willingness to pay from the consumers.

Did we solve the above problem correctly?

Think about (P, Q):

$$\max_{P,Q} P - cQ,$$
$$\text{s.t. } u(Q) \geq P.$$

Conditions: no resale, knowledge of the consumers, uniformity of the consumers(?).

11.1.2 2d Degree

Consumers are different,

M. cannot differentiate between them.

Offers a schedule of prices: depend on quantity of units purchased (non-linear prices, quantity discounts).

2 (rep) consumers: $u_1(x) < u_2(x)$, $u_1'(x) < u_2'(x)$.

Offer: (P_1, Q_1), (P_2, Q_2).

IR=Individual Rationality (participation) constraints:

$$\begin{aligned} u_1(Q_1) - P_1 &\geq 0, \\ u_2(Q_2) - P_2 &\geq 0. \end{aligned}$$

IC=Incentive Compatibility (self-selection) constraints:

$$\begin{aligned} u_1(Q_1) - P_1 &\geq u_1(Q_2) - P_2, \\ u_2(Q_2) - P_2 &\geq u_2(Q_1) - P_1. \end{aligned}$$

Monopolist:

$$\max_{P_i, Q_i} P_1 + P_2 - c(Q_1 + Q_2),$$
$$\text{s.t. } \textit{IR, IC}.$$

Generically, only two of these bind.

With assumptions above $u_1(Q_1) = P_1$ and $u_2(Q_2) - P_2 = u_2(Q_1) - P_1$.

11.1.3 3d Degree

Consumers are charged different prices, but a price is per unit.

Examples of categories: age, gender, status (student) (medical services?), income, race(?), separate markets.

$$p(y)\left[1-\frac{1}{|\epsilon(y)|}\right]=c'(y).$$

Therefore,

$$|\epsilon_1|<|\epsilon_2|\Rightarrow p_1>p_2.$$

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Other issues: bundling, creating demand, accumulating info on consumers,...