

Exam No. 3

Econ 14.05

Instructions: Answer the three following questions. Each one of them counts for 1/3. Each subsection counts 1/5.

1. Consumption and Saving in two period

a. This is straightforward,

$$\begin{aligned} c_1 + s_1 &= y_1 \\ c_2 &= y_2 + s_1(1+r) \\ s_1 &\geq -\lambda y_1 \end{aligned}$$

or which is the same,

$$\begin{aligned} c_1 + s_1 &= y_1 \\ c_2 &= y_2 + s_1(1+r) \\ s_1 &> -\lambda y_1 \end{aligned}$$

and

$$\begin{aligned} c_1 &= (1+\lambda)y_1 \\ c_2 &= y_2 + \lambda y_1(1+r) \\ s_1 &= -\lambda y_1 \end{aligned}$$

b. The intertemporal budget constraint (IBC) is then given by

$$\begin{aligned} c_1 + \frac{c_2}{1+r} &= y_1 + \frac{y_2}{1+r} \\ s_1 \geq -\lambda y_1 \quad y_1 - c_1 &\geq -\lambda y_1 \quad c_1 \leq (1+\lambda)y_1 = y^* \end{aligned}$$

So we have the standard IBC until the point where the borrowing constraint binds.

c. From the maximization problem we get

$$\frac{c_2}{c_1} = (1+r)\beta \Rightarrow c_2 = (1+r)\beta c_1.$$

Plugging this back into the IBC, we obtain:

$$c_1 + \beta c_1 = y_1 + \frac{y_2}{1+r}$$

Therefore,

$$\begin{aligned} c_1 &= \frac{1}{(1+\beta)} \left\{ y_1 + \frac{y_2}{1+r} \right\}, \\ s_1 &= \frac{\beta}{(1+\beta)} \left\{ y_1 - \frac{y_2}{1+r} \right\}, \end{aligned} \quad \#$$

d. The constraint binds when the amount the consumer desires to borrow on the first period is bigger than what he is allowed to borrow, or in other terms

$$s_1 < -\lambda y_1 \quad c_1 > (1+\lambda)y_1$$

which, substituting the expression for c_1 from c, gives us

$$\frac{1}{(1+\beta)} \left\{ y_1 + \frac{y_2}{1+r} \right\} < (1+\lambda)y_1$$

or

$$\beta < \frac{\frac{y_2}{1+r} - \lambda y_1}{(1+\lambda)y_1} = \beta^*.$$

If that is the case then

$$\begin{aligned} c_1 &= (1+\lambda)y_1 \\ s_1 &= -\lambda y_1 \end{aligned}$$

the consumer consumes as much as she cans in period 1.

e. Both things make more likely to hit the constraint. the reason is that the richer you are tomorrow and the less you like

consuming tomorrow, the more you want to consume today and thus the more you want to borrow.

2. Recessions and fiscal policy in an overlapping generations model

a.

i.

$$c_{1t} + s_{1t} + b_{1t+1} = w_t - T_t$$

$$c_{2t} = (s_{1t} + b_{1t+1})(1 + r_{t+1})$$

Therefore, the intertemporal budget constraint becomes:

$$c_{1t} + \frac{c_{2t+1}}{1+r} = w_t - T_t$$

ii. Under logarithmic utility, the individual spend a fraction of her income in consumption in each period. The fraction spent in x is given by the ratio of the coefficient multiplying $\log(x)$ to the sum of all coefficients. In this case, both log are multiplied by 1. Therefore, the individual spends 1/2 of its intertemporal income in c_{1t} , and 1/2 in c_{2t+1} .

$$c_{1t} = \frac{1}{2}(w_t - T_t)$$

$$s_{1t} = \frac{1}{2}(w_t - T_t) - b_{t+1}$$

b.

i.

$$w_t = F_L(K, L)$$

$$= (1 - \alpha) \left(\frac{K_t}{L_t} \right)^\alpha A_t^{(1-\alpha)}$$

$$= (1 - \alpha) A_t^{(1-\alpha)} k_t^\alpha$$

ii.

$$K_{t+1} = s_{1t} L_t$$

$$k_{t+1} = \frac{1}{(1+n)} \left(\frac{1}{2}(w_t - T_t) - b_{t+1} \right)$$

$$k_{t+1} = \frac{1}{(1+n)} \left(\frac{1}{2}((1 - \alpha) A_t^{(1-\alpha)} k_t^\alpha - T_t) - b_{t+1} \right)$$

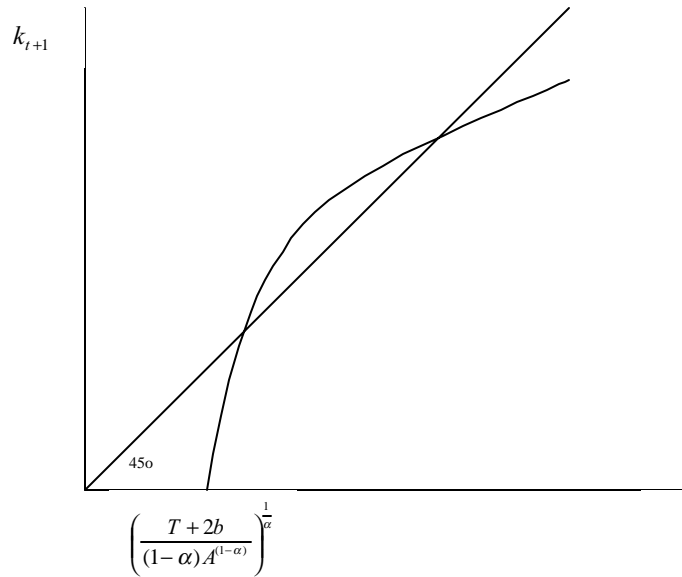
c.

i.

$$k^* = \frac{1}{2(1+n)} (1 - \alpha) A^{(1-\alpha)} (k^*)^\alpha - \frac{1}{(1+n)} \left(\frac{T}{2} + b \right)$$

#

ii.

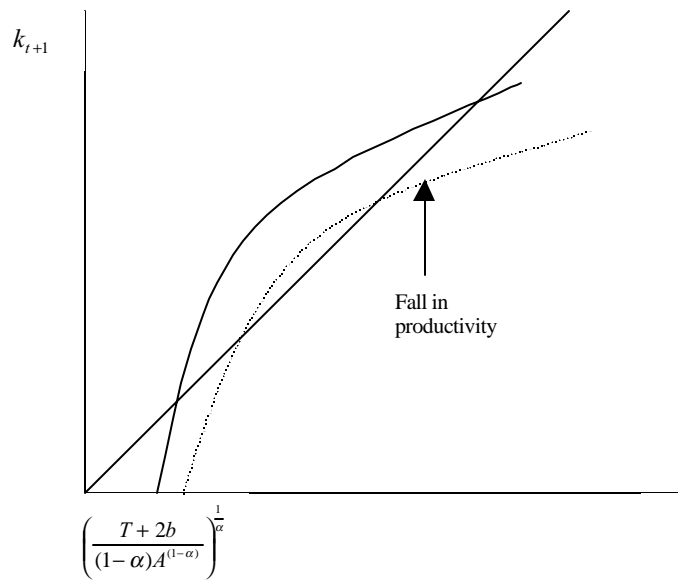


Economy has 3 equilibria: $k^* = 0$, k_0^* , k_1^* , only $k^* = 0$, and k_1^* are stable.

d.

i. Equilibrium level of capital falls.

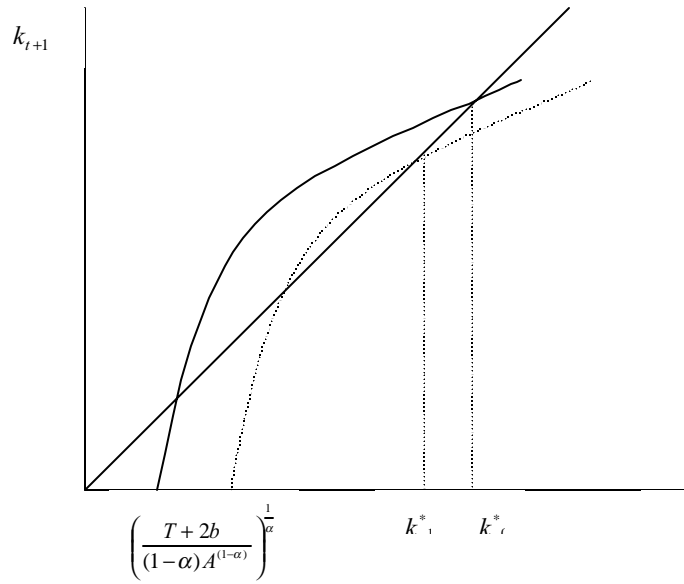
ii.



e.

i. A simple examination of equation (ref: 2bii), shows that this policy cannot increase the level of capital per-capita. The intuition is that, while the increase in bond emission reduces savings one by one, the fall in taxes increases savings only by the marginal propensity to save which is $1/2$ by one.

ii. In the graph we observe that as a result of the policy the equilibrium moves from k_0^* to k_1^* .



- iii. It does not seem very realistic. In fact, governments sometimes do fiscal policy by increasing expenditure and cutting taxes, which under this model will reduce the level of output.
- iv. The assumption of the model that is most responsible for this result is that government expenditure is “out of the economy”. In fact, the model assumes that government expenditure does not affect aggregate demand, nor the level of capital, and that government does not invest in capital goods. All of them are extreme assumptions.

3. Social Security:

- a.** Defined-Benefit: benefits are based on the characteristics of a person. Defined-Contribution: benefits are based on the amount contributed by a person. Social security today is a defined benefit program. The benefits are a fraction of lifetime taxable income. It is supplemented by 401(k) and IRA, which are defined contribution programs.
- b.** A funded retirement program invests the savings of the young and the retirement income is based on the returns to these savings. An unfunded retirement program is based on redistribution of income from the young to the old generation. It is a transfer. Social security is unfunded, except for the social security trust fund. Since 1977, the social security trust fund has gained importance, as the contributions to the trust fund were raised, without raising the benefits. In an OLG model, the main benefit of changing from unfunded to funded is an increase in the long run level of capital.
- c.** The Baby Boom is important for social security as it causes a financing problem. The working generation of the next 20 or 30 years has to contribute more per worker to finance a given benefit per retiree. This is because the proportion of old to young, (and hence from retired to working population) increases. With the current contribution rates and benefits, social security would go bankrupt within the next 20 years. President Bush's plans propose to solve the problem basically by cutting benefits (model 2) and also transferring money from general revenue (model 3).
- d.** Under President Bush's plan, the introduction of individual accounts worsens the budget problem of Social Security because the plans contemplate an implicit subsidy to individual accounts. This subsidy would have to be financed with transfers from general revenue.
- e.** Open question regarding the contents of the lecture and the Gruber's paper. Full credit assigned to any relevant and well reasoned answer.